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New phenomena in the standard no-scale supergravity model

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ABSTRACT

We revisit the no-scale mechanism in the context of the simplest no-scale supergravity extension of the Standard Model. This model has the usual five-dimensional parameter space plus an additional parameter $\xi_{3/2} \equiv m_{3/2}/m_{1/2}$. We show how predictions of the model may be extracted over the whole parameter space. A necessary condition for the potential to be stable is $\text{Str}\mathcal{M}^4 > 0$, which is satisfied if $\mathbf{m}_{3/2} \lesssim 2\mathbf{m}_{\tilde{q}}$. Order of magnitude calculations reveal a no-lose theorem guaranteeing interesting and potentially observable new phenomena in the neutral scalar sector of the theory which would constitute a “smoking gun” of the no-scale mechanism. This new phenomenology is model-independent and divides into three scenarios, depending on the ratio of the weak scale to the vev at the minimum of the no-scale direction. We also calculate the residual vacuum energy at the unification scale ($C_0 m_{3/2}^4$), and find that in typical models one must require $C_0 > 10$. Such constraints should be important in the search for the correct string no-scale supergravity model. We also show how specific classes of string models fit within this framework.

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1 Introduction

The search for a unified theory of everything has two boundaries. At one extreme the Standard Model matches flawlessly with experiment, and at the other extreme string theory promises quantum consistent unification of gravity with the fields of lower spin. In connecting these extremes, two vital clues have inspired major progress over the years: the gauge hierarchy problem, and the problem of vanishing cosmological constant. Global supersymmetry provides a first solution to the gauge hierarchy problem and ensures the cancellation of Λ^4 divergent one-loop contributions to the vacuum energy. However, global supersymmetry is not enough: we ultimately need local supersymmetry — supergravity. In addition, to simultaneously preserve the gauge hierarchy and respect the present experimental non-observation of sparticles, supergravity must be broken to a globally supersymmetric theory with soft breaking terms of order of the electroweak scale.

Supergravity is described in terms of two functions: the Kähler function (G) and the gauge kinetic function ($f_{\alpha\beta}$), plus the gauge and (hidden plus observable) matter content of the theory. Furthermore, the supersymmetry breaking scale (\tilde{m}) is not assured to be comparable to the electroweak scale. Moreover, in the process a large vacuum energy is usually generated ($\mathcal{O}(\tilde{m}^2 M_{Pl}^2) = \mathcal{O}(10^{-32} M_{Pl}^4)$). A very important exception to this typical and unsatisfactory situation occurs in a class of supergravity models with a distinct Kähler function endowed with some non-compact symmetries (*e.g.*, $SU(1, 1)$ or $SU(N, 1)$) [1, 2, 3]. This class of theories have three remarkable properties after supersymmetry breaking: (i) the minimum of the scalar potential is at zero vacuum energy [1], (ii) at the minimum there is one or more flat directions (moduli) which leave the gravitino mass undetermined [1, 2], and (iii) there are no large one-loop corrections to the vacuum energy ($\propto \text{Str}\mathcal{M}^2$) which would destroy the flat directions [2]. The first property implies that the vacuum energy is $\mathcal{O}(M_Z^4) = \mathcal{O}(10^{-64} M_{Pl}^4)$ or smaller. The second property provides a natural solution to the gauge hierarchy problem via the *no-scale* mechanism [4, 2], whereby the minimization of the electroweak potential determines the vacuum expectation value of the Higgs fields and of the moduli fields associated with the flat directions, thus determining the scale of supersymmetry breaking to be comparable to the electroweak scale. The third property ensures that the no-scale mechanism is stable under radiative corrections which would otherwise jeopardize the gauge hierarchy.

The above class of models were first proposed in the context of supergravity *per se*, and have been since found to be strongly supported by the low-energy effective theories from string [5, 6, 7, 8, 9, 10, 11]. In fact, in string model-building the class of “*string no-scale supergravities*” is much extended [8, 9] to include new kinds of moduli fields and more definite forms for the Kähler function [6, 10]. Moreover, in string models one can accommodate the usual non-perturbative (*e.g.*, gaugino condensation) supersymmetry breaking mechanism, as well as tree-level breaking via coordinate-dependent compactifications [8, 9]. All these features make string no-scale supergravity a very rich, interesting, and well motivated subset of all possible string supergravities. Furthermore, the three properties mentioned above are expected to

be very discriminating in the selection of phenomenologically appealing string vacua.

In this paper we explicitly solve the minimal no-scale supergravity extension of the Standard Model. We identify the minimal parameter space of this model and determine the other parameters by minimization with respect to the Higgs fields and the additional “*no-scale*” direction. A necessary condition for stability of the potential in the no-scale direction is $\text{Str}\mathcal{M}^4 > 0$, which requires $m_{3/2} \lesssim 2m_{\tilde{q}}$. The real and imaginary degrees of freedom corresponding to the additional *no-scale field* lead to new phenomenology in the neutral scalar sector. Calculation of the residual vacuum energy at the unification scale as a function of the soft-supersymmetry-breaking parameters leads to interesting constraints on the parameters of the model.

2 The standard no-scale supergravity model

Consider the simplest supergravity extension of the Standard Model. Numerous previous studies have identified the parameter space of this model, shown how to solve the model, and extracted its predictions as a function of this five-dimensional parameter space which can be taken as $(\tan\beta, m_t, m_{1/2}, \xi_0, \xi_A)$, with $\xi_0 \equiv m_0/m_{1/2}$ and $\xi_A \equiv A/m_{1/2}$. However, the naive construction of the one-loop effective potential [12] has a major flaw: it is not formally independent of the renormalization scale, although derivatives with respect to the various Higgs fields are. This problem was studied in Ref. [13], where a simple and well motivated ansatz for a Q -independent one-loop effective potential was proposed: one should subtract the field-independent contribution to the potential. That is, one should use the following expression

$$V_1 = V_0 + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) - V_1(0), \quad (1)$$

where V_0 is the usual RGE-improved tree-level Higgs potential, $V_1(0)$ is the field-independent contribution to V_1 , and the supertrace ($\text{Str} \mathcal{M}^n = \sum_j (-1)^{2j} (2j+1) \text{Tr} \mathcal{M}_j^n$) includes a term for the gravitino. Although the field-independent term and the contribution of the gravitino in the supertrace are irrelevant to minimization with respect to the Higgs fields, these two terms are crucial to dynamically determining supersymmetry breaking.

Since the field-independent contribution to V_1 could contain an unknown Q -independent piece, we parametrize it more generally as $C m_{3/2}^4$. The RGE satisfied by C is easily derived by demanding that V_1 be scale-independent to one-loop order (see Eq. (2.15) of Ref. [13])

$$\frac{dV_1}{dt} = \frac{dV_0}{dt} - \frac{1}{32\pi^2} \text{Str} \mathcal{M}^4 + m_{3/2}^4 \frac{dC}{dt} + (\text{“two-loop”}) = 0. \quad (2)$$

Moreover, the resulting relation must hold for all values of the fields. Taking the Higgs fields (h_1, h_2) to zero gives $V_0 = dV_0/dt = 0$, and the RGE follows

$$m_{3/2}^4 \frac{dC}{dt} = \frac{1}{32\pi^2} [\text{Str} \mathcal{M}^4]_{h_{1,2}=0}, \quad (3)$$

with (in the notation of Ref. [3])

$$\begin{aligned} [\text{Str}\mathcal{M}^4]_{h_{1,2}=0} = & 4(m_1^4 + m_2^4 + 2m_3^4) + 6(m_{U_3^c}^4 + m_{D_3^c}^4 + 2m_{Q_3}^4) + 12(m_{U^c}^4 + m_{D^c}^4 \\ & + 2m_Q^4) + 2(m_{E_3^c}^4 + 2m_{L_3}^4) + 4(m_{E^c}^4 + 2m_L^4) - 16M_3^4 - 6M_2^4 - 2M_1^4 \\ & - 8\mu^4 - 4m_{3/2}^4. \end{aligned} \quad (4)$$

Interestingly enough, there has been a recent effort to identify the value of C at the scale of supersymmetry breaking (C_0) as the remnant vacuum energy after supersymmetry breaking [14]. Moreover, C_0 is in principle calculable in specific string models and in specific supersymmetry breaking mechanisms. For example, in tree-level supersymmetry breaking via coordinate-dependent compactifications one has $C_0 \sim (n_B - n_F)$, where $n_{B(F)}$ is the number of massless bosonic (fermionic) degrees of freedom after supersymmetry breaking [15, 11].

The essence of the no-scale approach is that the gravitino mass is a function of the real part of an additional scalar field, $T_R \equiv \text{Re}(T)$, whose vev is undetermined at tree level, *i.e.*, there is a flat direction. We assume the following functional form

$$m_{3/2}^2 = \frac{\alpha \Lambda^{2+p}}{(T_R)^p}, \quad (5)$$

though it is interesting to consider departures from this. Here α is a dimensionless parameter and Λ is some appropriate mass scale. It is also possible to have $m_{3/2}$ depend on multiple flat directions, a possibility which is realized in many string examples. The gravitino mass is dynamically determined by the minimum of the loop-corrected potential with respect to T_R . In this section we derive some model-independent results by virtue of the fact that the zeroes of the first derivative of the potential with respect to T_R , and the sign of the second derivative are independent of α and p .

To implement the no-scale mechanism one can take one of two approaches: (i) a top-down approach, where C_0 is “given” and minimization of V_1 with respect to $m_{3/2}$ (*i.e.*, the no-scale mechanism) gives $m_{3/2}$; or (ii) a bottom-up approach, where one uses the no-scale mechanism to determine $C(M_Z)$ for a given value of $m_{3/2}$ and then obtains C_0 by RGE evolution. Since C_0 is in practice rather unknown, here we follow the bottom-up approach with the hope of finding constraints on the calculated value of C_0 (as a function of the usual soft-supersymmetry-breaking parameters), which should help guide string model builders in their quest for models with phenomenologically acceptable values of C_0 .

We now calculate the first and second derivatives of V_1 with respect to T_R . To proceed we scale out the $m_{3/2}$ dependence in V_1 (here $X \equiv m_{3/2}^2$)

$$V_1 = X^2 \hat{V}_0 + X^2 C + \frac{X^2}{64\pi^2} \text{Str}\hat{\mathcal{M}}^4 \left(\ln \frac{X\hat{\mathcal{M}}^2}{Q^2} - \frac{3}{2} \right), \quad (6)$$

where $\hat{V}_0 = V_0/X^2$, etc. Thus we obtain (see also Ref. [14])

$$\frac{\partial V_1}{\partial T_R} = -\frac{p}{T_R} X \frac{\partial V_1}{\partial X} = -\frac{p}{T_R} \left\{ 2V_1 + \frac{1}{64\pi^2} \text{Str}\mathcal{M}^4 \right\}, \quad (7)$$

and the no-scale condition $\frac{\partial V_1}{\partial T_R} = 0$ then implies

$$V_1 = -\frac{1}{128\pi^2} \text{Str}\mathcal{M}^4, \quad (8)$$

which allows one to determine $C = C(m_{3/2})$ in the bottom-up approach.¹ Is this a true minimum or just an extremum? A *necessary* condition is $\partial^2 V_1 / \partial T_R^2 > 0$ at the minimum. We obtain

$$T_R^2 \frac{\partial^2 V_1}{\partial T_R^2} = (p + 3p^2)X \frac{\partial V_1}{\partial X} - 2p^2 V_1 + \frac{p^2}{64\pi^2} \text{Str}\mathcal{M}^4, \quad (9)$$

and at the minimum

$$\left(T_R^2 \frac{\partial^2 V_1}{\partial T_R^2} \right)_{\text{minimum}} = \frac{p^2}{32\pi^2} \text{Str}\mathcal{M}^4. \quad (10)$$

Therefore, if $\text{Str}\mathcal{M}^4 > 0$, then the minimum in the moduli direction is stable, and the vacuum energy is negative.

The necessary condition for stability of the no-scale mechanism ($\text{Str}\mathcal{M}^4 > 0$) has an important consequence: it imposes an upper bound on $m_{3/2}$ as a function of the usual soft-supersymmetry-breaking parameters. If we define the ratios

$$\xi_{3/2} \equiv \frac{m_{3/2}}{m_{1/2}}, \quad \xi_0 \equiv \frac{m_0}{m_{1/2}}, \quad \xi_A \equiv \frac{A}{m_{1/2}}, \quad (11)$$

then the upper bound is on the parameter $\xi_{3/2}$. To get a rough estimate of this bound, we consider only the dominant contributions to $\text{Str}\mathcal{M}^4$, namely those from the three generations of (degenerate) squarks and the gravitino:

$$\text{Str}\mathcal{M}^4 \approx 6 \times 12m_{\tilde{q}}^4 - 4m_{3/2}^4 > 0. \quad (12)$$

Therefore

$$\frac{m_{3/2}}{m_{\tilde{q}}} < (18)^{1/4} \approx 2.06. \quad (13)$$

In practice we expect the factor to be slightly higher because of the neglected contributions to $\text{Str}\mathcal{M}^4$. Since to good approximation one can write $m_{\tilde{q}} \approx m_0 \sqrt{1 + c_{\tilde{q}}/\xi_0^2}$, with $c_{\tilde{q}} \sim 4 - 6$ an RGE-dependent constant, we also have

$$\frac{\xi_{3/2}}{\xi_0} \lesssim 2.06 \sqrt{1 + c_{\tilde{q}}/\xi_0^2}. \quad (14)$$

In addition to this necessary requirement for stability of the potential, the absence of negative mass eigenstates in the neutral scalar sector provides more stringent although model-dependent constraints which are analyzed in the next section.

¹Or equivalently $m_{3/2} = m_{3/2}(C_0)$ in the top-down approach.

3 New phenomenology: the neutral scalar sector

Turning to the Higgs mass spectrum, interesting new possibilities beyond the Supersymmetric Standard Model exist because of the new neutral scalar degrees of freedom corresponding to the flat direction(s). Although the analysis of the previous section was independent of the particular relation between $m_{3/2}$ and the flat direction(s), this relation enters the mass matrix of the real neutral scalars. For simplicity, we analyze these new possibilities assuming only one flat direction, but we believe the qualitative form of these results persist in models with more flat directions. The resulting phenomenology neatly divides into three cases, depending on the ratio $\gamma \equiv m_{3/2}/T_R$, and each case has interesting and potentially observable new phenomena. In the limit of exact CP conservation, the neutral real and imaginary degrees of freedom decouple. Since the potential is independent of $T_I = \text{Im}(T)$, this field is exactly massless and completely decouples, leaving the spectrum of imaginary Higgs degrees of freedom unchanged. However, T_R must be included in a 3×3 mass matrix along with the usual two real Higgs degrees of freedom.

Using the relation between $m_{3/2}$ and one flat direction in Eq. (5), and evaluating the derivatives with respect to T_R , gives the following 3×3 mass matrix for the neutral real scalar degrees of freedom

$$\frac{1}{2} \begin{pmatrix} \partial^2 V_1 / \partial h_1 \partial h_1 & \partial^2 V_1 / \partial h_1 \partial h_2 & \frac{p}{64\pi^2} \frac{1}{T_R} \partial \text{Str} \mathcal{M}^4 / \partial h_1 \\ \partial^2 V_1 / \partial h_2 \partial h_1 & \partial^2 V_1 / \partial h_2 \partial h_2 & \frac{p}{64\pi^2} \frac{1}{T_R} \partial \text{Str} \mathcal{M}^4 / \partial h_2 \\ \frac{p}{64\pi^2} \frac{1}{T_R} \partial \text{Str} \mathcal{M}^4 / \partial h_1 & \frac{p}{64\pi^2} \frac{1}{T_R} \partial \text{Str} \mathcal{M}^4 / \partial h_2 & \frac{2p}{64\pi^2} \frac{1}{T_R^2} \text{Str} \mathcal{M}^4 \end{pmatrix}. \quad (15)$$

The analysis of this matrix is simplified by making a change of basis on the Higgs fields to introduce some zero entries

$$\begin{pmatrix} a & b & 0 \\ b & c & d \\ 0 & d & e \end{pmatrix}, \quad (16)$$

where $a, b, c = \mathcal{O}(m_{3/2}^2)$, $d = \mathcal{O}(\gamma m_{3/2}^2)$, and $e = \mathcal{O}(\gamma^2 m_{3/2}^2)$. In the limits $\gamma \gg 1$ or $\gamma \ll 1$, the form of the eigenvectors can be identified as

$$\begin{pmatrix} \cos \phi \\ \sin \phi \\ \epsilon_1 \end{pmatrix}, \quad \begin{pmatrix} -\sin \phi \\ \cos \phi \\ \epsilon_2 \end{pmatrix}, \quad \begin{pmatrix} \epsilon_3 \\ \epsilon_4 \\ 1 \end{pmatrix}, \quad (17)$$

where $|\epsilon_{1,2,3,4}| \ll 1$. In these two limits there is little mixing between the Higgs and T_R fields.

In the $\gamma \ll 1$ limit we obtain the usual Higgs-boson masses with small $\mathcal{O}(\gamma^2 m_{3/2}^2)$ corrections to the squared masses. To first order we obtain $m_{T_R}^2 = e - ad^2/(ca - b^2) = \mathcal{O}(\gamma^2 m_{3/2}^2)$, and there is a new, very light scalar. Assuming that the usual 2×2 Higgs mass matrix has a positive determinant, $m_{T_R}^2 > 0$ is equivalent to the whole mass

matrix having a positive determinant. In the $\gamma \gg 1$ limit the Higgs masses are given to first order by

$$m_{H_R}^2 = \frac{1}{2} \left\{ c + a - \frac{d^2}{e} \pm \sqrt{\left(\frac{d^2}{e} - c + a \right)^2 + 4b^2} \right\}, \quad (18)$$

which depart from their usual value in models without the T_R field because of the new d^2/e terms. Again, assuming the usual 2×2 Higgs mass matrix has a positive determinant, $m_{H_R}^2 > 0$ is equivalent to the whole mass matrix having a positive determinant. Note that the resulting Higgs masses are independent of γ , since $d^2/e = \mathcal{O}(m_{3/2}^2)$. The mass of T_R is $e \sim \mathcal{O}(\gamma^2 m_{3/2}^2) \gg m_{3/2}^2$. In the case where $\gamma \sim 1$, the mass eigenstates are general mixtures of the Higgs and T_R fields, and must be analyzed as a function of parameter space.

We reach the very interesting conclusion that no-scale supergravity always results in phenomenology beyond its global supersymmetric counterpart because of the fields associated with the flat direction. The real neutral scalar phenomenology falls into three cases. For $\gamma \ll 1$, there is an additional scalar with $m^2 \sim \mathcal{O}(\gamma^2 m_{3/2}^2)$ and unaltered Higgs predictions. For $\gamma \gg 1$ the usual predictions for the Higgs masses are altered. For $\gamma \sim 1$, there is a rich new phenomenology arising from a mass matrix significantly mixing the H_R and T_R fields which has detailed dependence on the parameter space of the model. This ‘no-lose’ situation offers hope for experimentally verifying no-scale supergravity. In addition, the distinct phenomenological properties of the three cases provides an experimental handle on the parameter γ , and thereby information about the Kähler potential.

4 Numerical investigations

The unknown parameter space of our model may be taken as $(\tan \beta, m_t, m_{1/2}, \xi_0, \xi_A, \xi_{3/2})$. Given these parameters the Higgs mixing terms μ and B can be calculated from the usual minimization conditions [13]. The parameter C can be calculated from the requirement that the potential is a minimum with respect to $m_{3/2}$. Although this may be accomplished by taking derivatives numerically, a more efficient (and accurate) method results from setting Eqs. (1) and (8) equal and solving for $C_{1/2} \equiv C \xi_{3/2}^4$.

$$C_{1/2} m_{1/2}^4 = - \left[V_0 + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) + \frac{1}{128\pi^2} \text{Str} \mathcal{M}^4 \right]. \quad (19)$$

The problem may be further simplified by separating out the contribution (in the supertraces) of the gravitino to $C_{1/2}$ from the other contributions,

$$C_{1/2} = C_{1/2}^g + \tilde{C}_{1/2}. \quad (20)$$

Note that $\tilde{C}_{1/2}$ depends on the five parameters $(\tan \beta, m_t, m_{1/2}, \xi_0, \xi_A)$ and is independent of $\xi_{3/2}$. On the other hand, $C_{1/2}^g$ depends only on $(\xi_{3/2}, m_{1/2})$ and is independent

of the other four parameters:

$$C_{1/2}^g = \frac{\xi_{3/2}^4}{16\pi^2} \left(\ln \frac{\xi_{3/2}^2 m_{1/2}^2}{M_Z^2} - 1 \right). \quad (21)$$

Next, consider the scaling of $C_{1/2}$ (from M_Z to the unification scale M_U) which obeys the same RGE as C , Eq. (3), with $m_{1/2}$ replacing $m_{3/2}$

$$C_{1/2}(M_U) = C_{1/2}(M_Z) + \Delta C_{1/2}. \quad (22)$$

Since the RGE is linear, it can also be separated into a part due to the contribution of the gravitino and a part due to all the other particles, and we can write

$$\Delta C_{1/2} = \Delta C_{1/2}^g + \Delta \tilde{C}_{1/2}, \quad (23)$$

with a simple expression for $\Delta C_{1/2}^g$

$$\Delta C_{1/2}^g = \frac{\xi_{3/2}^4}{8\pi^2} \ln \frac{M_U}{M_Z}. \quad (24)$$

We have given the explicit analytic expressions for $C_{1/2}^g$ and $\Delta C_{1/2}^g$, whereas $\tilde{C}_{1/2}$ and $\Delta \tilde{C}_{1/2}$ must be calculated numerically as a function of the five-dimensional parameter space $(\tan \beta, m_t, m_{1/2}, \xi_0, \xi_A)$. However, within the phenomenologically viable area of this five-dimensional parameter space, we find the results nearly independent of $\tan \beta$, m_t , and ξ_A , with a small dependence on $m_{1/2}$. Therefore, we present our final results for $C_0 = C(M_U)$ in the $(\xi_0, \xi_{3/2})$ plane shown in Figure 1. The particular plot shown is for $m_t = 150$ GeV, $\tan \beta = 2$, $\xi_A = 0$, and $m_{1/2} = 100$ GeV although changing these variables shifts the contours only slightly.

Note that the stability of the potential puts an upper bound on $\xi_{3/2}$ which is well approximated by Eq. (14). If there were some upper bound on the magnitude of C_0 from naturalness or string considerations, this would give a lower bound on $\xi_{3/2}$. For example, if we require $|C_0| < 10$, then for $\xi_0 = 0$, $1.2 < \xi_{3/2} < 4.6$. For $\xi_0 = 4$ the bound becomes $4.1 < \xi_{3/2} < 9.9$. Turning things around, typical supersymmetry breaking scenarios entail $m_0 = m_{3/2}$ (*i.e.*, $\xi_{3/2} = \xi_0$) [16] or $m_{1/2} = m_{3/2}$ (*i.e.*, $\xi_{3/2} = 1$) [15]. In both cases Fig. 1 shows that $C_0 > 10$ is required. These types of bounds are very interesting in the context of explicit predictions for the various ξ 's in string models, and will be even more interesting in the context of sparticle spectroscopy [17].

5 Simple models

Assuming that the only fundamental scales in the theory are M_{Pl} and $m_{3/2}$, dimensional arguments for the simplest relation between T_R and $m_{3/2}$ give

$$m_{3/2}^2 = \frac{\alpha \kappa^{-p-2}}{(T_R)^p}, \quad (25)$$

where $\kappa = \sqrt{8\pi}/M_{Pl}$ is the gravitational coupling. This relation is the analogue of Eq. (5) for $\Lambda \sim M_{Pl}$. From this relation we obtain

$$\gamma = \frac{m_{3/2}}{T_R} \sim \frac{1}{\alpha^{1/p}} \left(\frac{m_{3/2}}{M_{Pl}} \right)^{1+2/p}. \quad (26)$$

Thus, the three scenarios for γ are related to the functional dependence of the gravitino mass: $\gamma \gg 1$ implies $\alpha \ll (m_{3/2}/M_{Pl})^{p+2}$, $\gamma \ll 1$ implies $\alpha \gg (m_{3/2}/M_{Pl})^{p+2}$, and $\gamma \sim 1$ implies $\alpha \sim (m_{3/2}/M_{Pl})^{p+2}$.

We now consider a choice for the Kähler function which possesses the three properties described in the introduction. In string models built within the free-fermionic formulation, one can show that the untwisted sector fields split up into three sets, each with a separate contribution to the Kähler function. In the simplest models these contributions can be written as [8, 10]

$$G = -\ln(S + \bar{S}) - \sum_{A=1}^3 \ln Y_A + K_{TS} + \ln |W|^2, \quad (27)$$

with

$$Y_A = 1 - \sum_{i_A} \alpha_{i_A} \bar{\alpha}_{i_A} + \frac{1}{4} \sum_{i_A} (\alpha_{i_A} \alpha_{i_A})(\bar{\alpha}_{i_A} \bar{\alpha}_{i_A}), \quad (28)$$

K_{TS} the twisted sector contribution, and W the superpotential. To make things simple we will neglect the second and third sets (*i.e.*, set $\alpha_{i_{2,3}} = 0$) as well as the twisted sector contribution. Also, we will only consider three fields in the first set. This approximation should suffice for our present purposes. Through an analytic field redefinition [8], our simplified Kähler function reduces to

$$G = -\ln(S + \bar{S}) - \ln[(U + \bar{U})(T + \bar{T}) - (\phi + \bar{\phi})^2] + \ln |W(\phi)|^2, \quad (29)$$

where T and U are the moduli fields, and ϕ is a charged matter field. The scalar potential is then

$$V = e^G \left\{ -(\phi + \bar{\phi})(\partial_\phi \ln W + \partial_{\bar{\phi}} \ln \bar{W}) + \frac{1}{2} \left[(U + \bar{U})(T + \bar{T}) + (\phi + \bar{\phi})^2 \right] |\partial_\phi \ln W|^2 \right\}, \quad (30)$$

which is not obviously positive semi-definite. Considering minima which preserve the gauge symmetry, *i.e.*, with $\langle \phi \rangle = \langle \bar{\phi} \rangle = 0$, the potential reduces to

$$V_{\langle \phi \rangle = \langle \bar{\phi} \rangle = 0} = \frac{1}{2} \frac{|W|^2}{(S + \bar{S})} |\partial_\phi \ln W|^2, \quad (31)$$

which is positive semi-definite. Therefore the minima, which occur for $\langle \partial_\phi \ln W \rangle = 0$, have zero vacuum energy. Moreover, the potential is manifestly T - and U -independent, *i.e.*, we have two flat directions.² The gravitino mass is given by

$$m_{3/2}^2 = \frac{\langle |W|^2 \rangle}{\langle (S + \bar{S})(U + \bar{U})(T + \bar{T}) \rangle} = \frac{g^2 \langle |W|^2 \rangle}{\langle (U + \bar{U})(T + \bar{T}) \rangle}, \quad (32)$$

²The value of S is also undetermined. This vev could be fixed by giving W and S -dependence.

and is undetermined because of the T and U flat directions. The goldstino is a to-be-determined linear combination of \tilde{S} , \tilde{T} , and \tilde{U} . Whether the third property (*i.e.*, $\text{Str}\mathcal{M}^2 = 0$) is satisfied or not depends on the mechanism for supersymmetry breaking [11]. In tree-level breaking via coordinate-dependent compactifications one has $W = w + \dots$, with the limit $W \rightarrow w$ for $\phi \rightarrow 0$, and w a constant of order 1. The $\text{Str}\mathcal{M}^2 = 0$ condition then imposes a non-trivial constraint on the number of fields belonging to each of the three sets of the theory [11]. Writing $m_{3/2}^2 = g^2 w^2 / R^2$, with $R^2 = (U + \bar{U})(T + \bar{T})\kappa^4$ the “radius” of the compactified dimension, it is clear that $m_{3/2} \sim 1 \text{ TeV}$ requires $R^{-1} \sim 1 \text{ TeV}$. In this “decompactification” limit there is a tower of Kaluza-Klein string massive states which can be as light as a few TeV and have distinct experimental signatures [15].

Let us take $T = U$, which then gives $p = 2$ and $\alpha = g|W|$ in Eq. (25), and therefore $\gamma \sim \alpha^{-1/2} (m_{3/2}/M_{Pl})^2$. If $\alpha \sim 1$, as expected in coordinate-dependent compactifications, then $\gamma \ll 1$. On the other hand, if $\alpha \ll 1$, as expected in gaugino condensation models, the $\gamma \sim 1$ may be possible. More possibilities may exist in more general supersymmetry breaking scenarios.

6 Conclusions

The solution to the gauge hierarchy problem via the no-scale mechanism selects a special class of string-derived supergravities. We have studied some simple examples of these very appealing models and have shown that the no-scale mechanism leads to a stable minimum of the electroweak potential for large regions of the soft-supersymmetry-breaking parameter space. Moreover, this stability requirement can be neatly encoded in the upper bound $m_{3/2} \lesssim 2m_{\tilde{q}}$. The no-scale mechanism also entails mixing of the Higgs fields with the moduli, which could be experimentally observable in some supersymmetry breaking scenarios. These two results appear to be *unique* to the no-scale mechanism and, if verified experimentally, would constitute the “smoking gun” of no-scale supergravity. We are in the process of addressing the laboratory and cosmological implications of such no-scale supergravity properties. We have also studied the dependence on the soft-supersymmetry-breaking parameters of the residual vacuum energy at the unification scale ($C_0 m_{3/2}^4$). We find that in typical models one must require $C_0 > 10$. Our results should be useful to string model builders searching for string no-scale supergravity models with phenomenologically viable values of C_0 .

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References

- [1] E. Cremmer, S. Ferrara, C. Kounnas, and D. V. Nanopoulos, Phys. Lett. B **133** (1983) 61.
- [2] J. Ellis, C. Kounnas, and D. V. Nanopoulos, Nucl. Phys. B **241** (1984) 406 and Nucl. Phys. B **247** (1984) 373.
- [3] For a review see A. B. Lahanas and D. V. Nanopoulos, Phys. Rep. **145** (1987) 1.
- [4] J. Ellis, A. Lahanas, D. V. Nanopoulos, and K. Tamvakis, Phys. Lett. B **134** (1984) 429.
- [5] E. Witten, Phys. Lett. B **155** (1985) 151.
- [6] I. Antoniadis, J. Ellis, E. Floratos, D. V. Nanopoulos, and T. Tomaras, Phys. Lett. B **191** (1987) 96; S. Ferrara, L. Girardello, C. Kounnas, and M. Petracci, Phys. Lett. B **193** (1987) 368 and Phys. Lett. B **194** (1987) 358.
- [7] S. Ferrara, C. Kounnas, M. Petracci, and F. Zwirner, Phys. Lett. B **194** (1987) 366.
- [8] S. Ferrara, C. Kounnas, M. Petracci, and F. Zwirner, Nucl. Phys. B **318** (1989) 75.
- [9] M. Petracci and F. Zwirner, Nucl. Phys. B **326** (1989) 162.
- [10] J. L. Lopez, D. V. Nanopoulos, and K. Yuan, Phys. Rev. D **50** (1994) 4060.
- [11] S. Ferrara, C. Kounnas, and F. Zwirner, CERN-TH.7192/94 (hep-th/9405188).
- [12] C. Kounnas, A. Lahanas, D. V. Nanopoulos, and M. Quirós, Nucl. Phys. B **236** (1984) 438, G. Gamberini, G. Ridolfi, and F. Zwirner, Nucl. Phys. B **331** (1990) 331.
- [13] S. Kelley, J. L. Lopez, D. V. Nanopoulos, H. Pois, and K. Yuan, Nucl. Phys. B **398** (1993) 3.
- [14] C. Kounnas, I. Pavel, and F. Zwirner, CERN-TH.7185/94 (hep-ph/9406256).
- [15] I. Antoniadis, Phys. Lett. B **246** (1990) 377; I. Antoniadis, C. Muñoz, and M. Quiros, Nucl. Phys. B **397** (1993) 515; I. Antoniadis and K. Benakli, Phys. Lett. B **326** (1994) 69 and Phys. Lett. B **331** (1994) 313.
- [16] L. Ibáñez and D. Lüst, Nucl. Phys. B **382** (1992) 305; V. Kaplunovsky and J. Louis, Phys. Lett. B **306** (1993) 269; A. Brignole, L. Ibáñez, and C. Muñoz, Nucl. Phys. B **422** (1994) 125.
- [17] A. Faraggi, J. Hagelin, S. Kelley, and D. V. Nanopoulos, Phys. Rev. D **45** (1992) 3272.

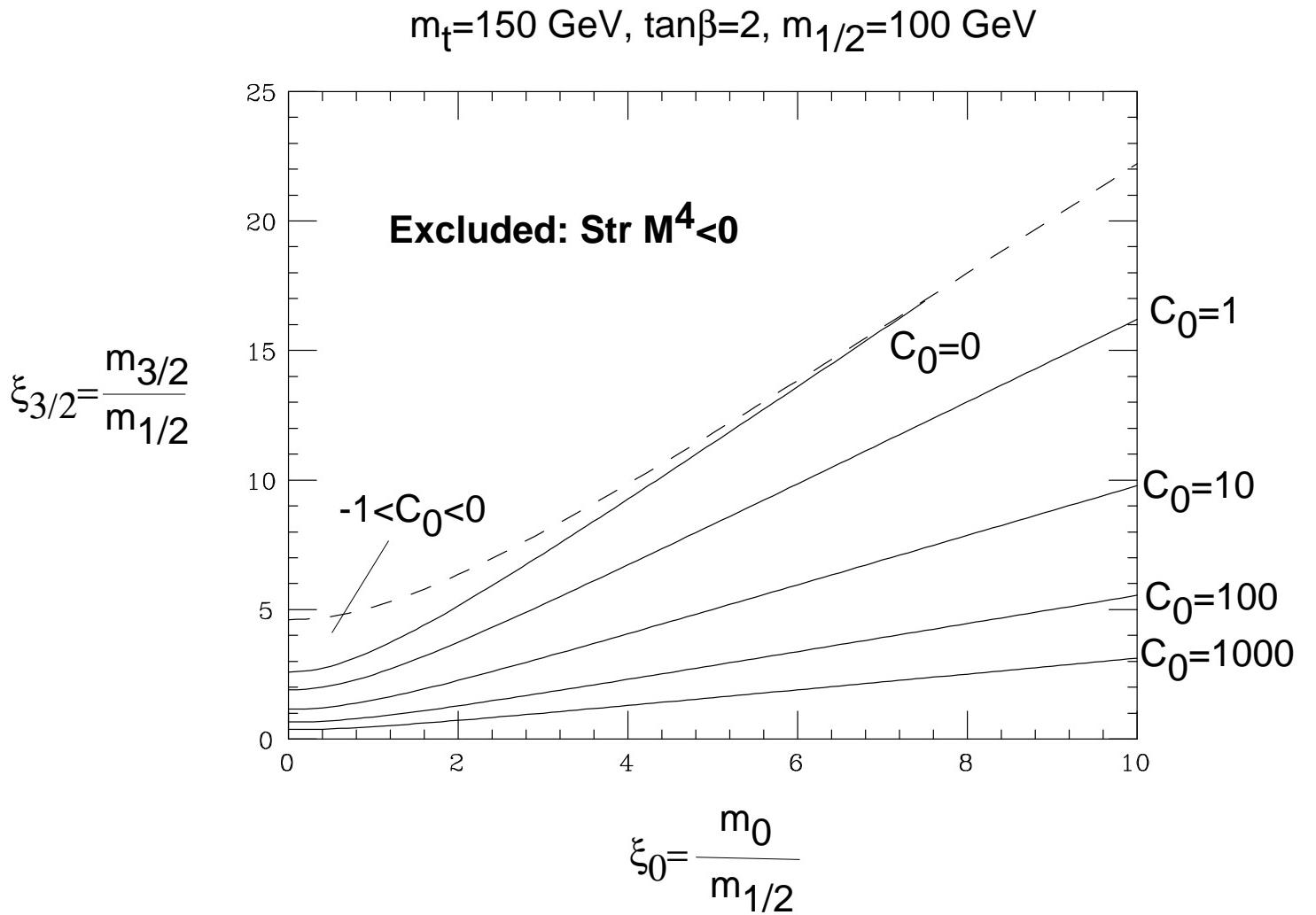


Figure 1: Contours of constant values of $C_0 = C(M_U)$ in the $(\xi_0, \xi_{3/2})$ plane. The dependence on $m_{1/2}$ is very mild; m_t and $\tan\beta$ are also immaterial. The area above the dashed line is excluded by the stability of the no-scale mechanism which requires $\text{Str}\mathcal{M}^4 > 0$.

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